

$$\int u dv = uv - \int v du$$

Choose u to be the first part appearing in the acronym:

L ogarithmic

I nverse Trig

A lgebraic

T rigonometric

E xponential

Ex:

$$\int \underbrace{x}_u \underbrace{e^x dx}_{dv}$$

algebraic exponential

Ex: $\int \underbrace{x}_{\text{alg}} \underbrace{\cos x dx}_{\text{trig}}$ $\left(\begin{array}{ll} u = x & dv = \cos x dx \\ du = dx & v = \sin x \end{array} \right)$

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x - (-\cos x + C)$$
$$= \boxed{x \sin x + \cos x + C}$$

$$\underline{\text{Ex:}} \int x \ln x \, dx \quad \left(\begin{array}{ll} u = \ln x & dv = x \, dx \\ du = \frac{1}{x} \, dx & v = \frac{1}{2} x^2 \end{array} \right)$$

$$= \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} \, dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x \, dx$$

$$= \boxed{\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C}$$

$$\underline{\text{Ex:}} \int x^2 e^x \, dx \quad \left(\begin{array}{ll} u = x^2 & dv = e^x \, dx \\ du = 2x \, dx & v = e^x \end{array} \right)$$

$$= x^2 e^x - \int 2x e^x \, dx \quad \left(\begin{array}{ll} u = 2x & dv = e^x \, dx \\ du = 2 \, dx & v = e^x \end{array} \right)$$

$$= x^2 e^x - (2x e^x - \int 2 e^x \, dx)$$

$$= \boxed{x^2 e^x - 2x e^x + 2e^x + C}$$

$$\underline{\text{Ex:}} \int \ln x \, dx \quad \left(\begin{array}{ll} u = \ln x & dv = dx \\ du = \frac{1}{x} \, dx & v = x \end{array} \right)$$

$$= x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - \int 1 \, dx$$

$$= \boxed{x \ln x - x + C}$$

$$\underline{\text{Ex:}} \int \underline{\arctan x} dx \quad \left(\begin{array}{l} u = \arctan x \quad dv = dx \\ du = \frac{1}{1+x^2} dx \quad v = x \end{array} \right)$$

$$= x \arctan x - \int \frac{x}{1+x^2} dx \quad \left(\begin{array}{l} u = 1+x^2 \\ du = 2x dx \\ \hookrightarrow \frac{1}{2} du = x dx \end{array} \right)$$

$$= x \arctan x - \frac{1}{2} \int \frac{1}{u} du$$

$$= x \arctan x - \frac{1}{2} \ln|u| + C$$

$$= \boxed{x \arctan x - \frac{1}{2} \ln|1+x^2| + C}$$

$$\underline{\text{Ex:}} \int \arcsin x dx \quad \left(\frac{d}{dx} [\arcsin x] = \frac{1}{\sqrt{1-x^2}} \right)$$

$$\left(\begin{array}{l} u = \arcsin x \quad dv = dx \\ du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x \end{array} \right)$$

$$\int \arcsin x dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx \quad \left(\begin{array}{l} u = 1-x^2 \\ du = -2x dx \end{array} \right)$$

$$= x \arcsin x + \frac{1}{2} \int \frac{1}{\sqrt{u}} du = x \arcsin x + \frac{1}{2} \int u^{-1/2} du$$

$$= x \arcsin x + u^{1/2} + C = \boxed{x \arcsin x + \sqrt{1-x^2} + C}$$

$$\underline{\text{Ex:}} \int e^x \sin x \, dx \quad \left(\begin{array}{l} u = \sin x \quad dv = e^x \, dx \\ du = \cos x \, dx \quad v = e^x \end{array} \right)$$

$$= e^x \sin x - \int e^x \cos x \, dx \quad \left(\begin{array}{l} u = \cos x \quad dv = e^x \, dx \\ du = -\sin x \, dx \quad v = e^x \end{array} \right)$$

$$= e^x \sin x - \left(e^x \cos x - \int -e^x \sin x \, dx \right)$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$\underbrace{\int e^x \sin x \, dx}_{+ \int e^x \sin x \, dx} = e^x \sin x - e^x \cos x - \underbrace{\int e^x \sin x \, dx}_{+ \int e^x \sin x \, dx}$$

$$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x + C$$

$$\hookrightarrow \boxed{\int e^x \sin x \, dx = \frac{1}{2} (e^x \sin x - e^x \cos x) + C}$$